A taxonomy of learning dynamics in 2×2 games

Marco Pangallo[⋆,1,2], James BT Sanders[3], Tobias Galla[3], and J. Doyne Farmer[1,2,4]

[1]Institute for New Economic Thinking at the Oxford Martin School, University of Oxford, Oxford OX26ED, UK

[2]Mathematical Institute, University of Oxford, Oxford OX13LP, UK

[3]Theoretical Physics, School of Physics and Astronomy, University of Manchester, Manchester M139PL, UK

[4]Santa Fe Institute, Santa Fe, NM 87501, US

Keywords: Behavioural Game Theory, EWA Learning, Convergence, Equilibrium, Chaos

How do players coordinate on specific profiles of strategies in non-cooperative games, and why should they coordinate on an equilibrium profile? If the game is simple or one-shot, a reasonable explanation is provided by strategic thinking and introspection. Another justification, which is more generally valid in complicated and repeated games, is learning and interaction. However, as it is fairly well known since the contribution of Shapley (1964), the learning dynamics may fail to converge to an equilibrium. This questions the validity of equilibrium thinking in game theory: at least in some contexts, strategic interactions might be governed by adaptation to an ever-changing environment, rather than by rational and fully-informed decision making.

The literature has faced the dilemma about the convergence of the learning dynamics to Nash Equilibria (NE) in several ways. Most theoretical work has identified classes of games and learning algorithms in which the dynamics succeeds to converge; some authors provided counter-examples in which learning would not converge. Galla and Farmer (2013) investigated the typical features of the learning dynamics that yielded unstable behaviour, showing that chaos could be seen in a significant portion of the parameter space. In order to shed light on the mechanisms behind (non-)convergence, this work investigates the drivers of instability in the simplest possible non-trivial setting, that is generic 2-person, 2-strategy normal form games. We study a slightly simplified version of Experience-Weighted Attraction (EWA), which is general enough to encompass both reinforcement and belief learning and has been shown to be in accord with experimental data (Camerer and Ho, 1999). We exhaustively characterize the parameter space of EWA learning, for any payoff matrix, and we understand the generic properties that imply convergent or non-convergent behaviour.

Irrational choice and lack of payoff incentives imply convergence to a mixed strategy in the centre of the strategy simplex, possibly far from the NE. The players simply randomize between their possible moves. In the opposite case, in which the players quickly modify their strategies to improve their payoff, the behaviour depends on the payoff matrix: (i) a strong discrepancy between the pure strategies describes dominance-solvable games, which show convergence to a unique fixed point close to the NE; (ii) a substantial difference between the diagonal and the antidiagonal elements describes coordination games, with multiple stable fixed points corresponding to the NE; (iii) a cycle in beliefs defines discoordination games, which commonly yield limit cycles or chaos. Lack of convergence is driven by the fact that the players adapt too quickly to the moves of their opponent. We find such a taxonomy of the learning dynamics by looking at relevant combinations of parameters, which naturally emerge from the mathematical analysis. Figure 1 illustrates our approach and provides a qualitative characterization of the parameter space.

From a methodological point of view, the main novelty is that we fully analyse EWA by employing tools from dynamical systems theory, and provide explicit thresholds that define

[*]Corresponding author: marco.pangallo@maths.ox.ac.uk

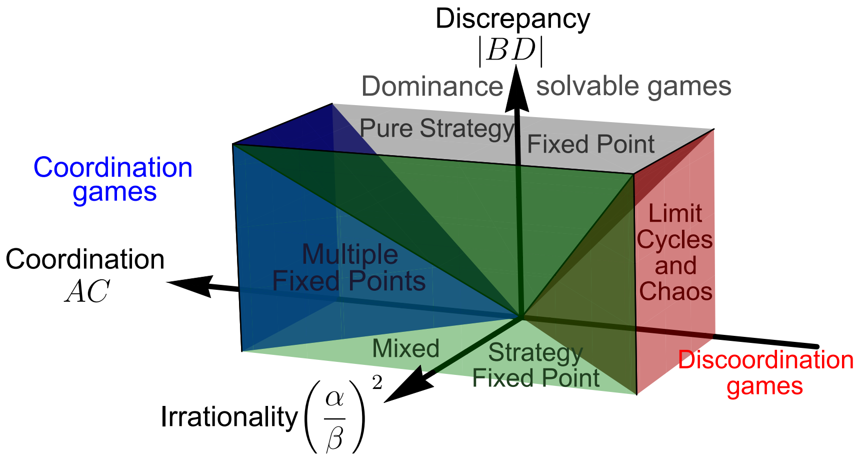

Figure 1: Qualitative characterization of the parameter space. *Irrationality* refers to the intrinsic noise in the learning algorithm. *Coordination* is the relative strength of the diagonal elements of the payoff matrix; if negative, the players try and coordinate on different profiles of pure strategies. *Discrepancy* is the relative strength of a pure strategy compared to the other one. These parameters characterize the learning dynamics and relate to specific classes of 2×2 games.

the onset of instability. Moreover, we find an emerging taxonomy of the learning dynamics, without focusing on specific classes of games *ex-ante*, but classifying them *ex-post* based on their dynamical properties. We are the first to find chaotic behaviour in a 2-dimensional strategy space with reinforcement and belief learning algorithms.

The ultimate goal of this line of research is to test whether learning converges in experiments. Is our learning algorithm representative of how players learn in reality, and do limit cycles or chaos in the learning dynamics play a role in the real world? It is likely that the players would change their learning strategy as the game evolves, implying that they learn how to learn (Stahl (1996) considered a model of *rule learning*). Our analysis suggests that limit cycles and chaos may theoretically be observed as long as the players are willing to quickly switch their moves to improve their payoff, independently of the reason why they behave so. A property of the cycling behaviour, as opposed to the convergence to a mixed strategy equilibrium, is the slower decay in the autocorrelation function of the strategies chosen by each player. In the language of time series, the sequence of moves by each player exhibits *persistence*. This is a precise theoretical prediction that can be tested against data on experimental learning of discoordination games. Only experiments will be able to provide a definitive answer on the dilemma about the convergence of learning in games.

Bibliography

Camerer, C. and Ho, T. (1999). *Experience-weighted attraction learning in normal form games*. In ≪Econometrica≫, vol. 67(4), pp. 827–874.

Galla, T. and Farmer, J. D. (2013). *Complex dynamics in learning complicated games*. In ≪Proceedings of the National Academy of Sciences≫, vol. 110(4), pp. 1232–1236.

Shapley, L. S. (1964). *Some topics in two-person games*. In ≪Advances in game theory, Annals of Mathematical Studies≫, vol. 52, pp. 1–29.

Stahl, D. O. (1996). *Boundedly rational rule learning in a guessing game*. In ≪Games and Economic Behavior≫, vol. 16(2), pp. 303–330.